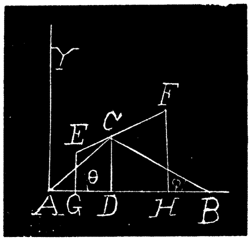
$$\cot\theta=\frac{b^2+c^2-1}{2c},\ \cot\varphi=\frac{c^2-b^2+1}{2bc},\ \cot\theta+\cot\varphi=\frac{AD+DB}{DC}=a/DC,$$

$$\therefore\ DC=\frac{2abc}{b^3-b^2-b+bc^2+c^2+1}.$$



DC must be $>EG$ and $<HF$.

Let $AG=m$, $EG=n$, $AH=h$, $HF=k$.

Then $b=\dfrac{hn-km}{ak+hn-an-km}$, $c=\dfrac{a(h-m)}{ak+hn-an-km}$,

b must be less than unity or EF may intersect AB.

Let $m=n=1$, $h=a=10$, $k=4$. $\therefore\ b=\frac{1}{6}$, $c=\frac{5}{2}$, $DC=\frac{36}{35}$.

$\cot\varphi=\frac{131}{15}$, $\cot\theta=\frac{19}{18}$. $\therefore\ \theta=43°\ 27'\ 6''$, $\varphi=6°\ 31'\ 56''$.

$AD=DC\cot\theta=\frac{38}{35}$.

89. Proposed by B. F. FINKEL, A. M., M. Sc., Professor of Mathematics and Physics, Drury College, Springfield, Mo.

Describe a circle tangent to three given circles. [From *Chauvenet's Geometry*, page 318, ex. 213.

Solution by G. B. M. ZERR, A. M., Ph. D., President and Professor of Mathematics, The Russell College, Lebanon, Va.; and F. R. HONEY, Ph. B., Instructor in Mathematics in Trinity College, New Haven, Conn.

In figure 2 let L, M, N be the circles radii a, b, c.

With M as center and radius $b-a$ describe a circle, also with N as center and radius $c-a$ describe a circle. Draw a circle through L tangent to the circles last described at T, S then the center of this circle is the center of one of the tangent circles. Similarly we can find seven other tangent circles.

Of the eight circles one is tangent to the three circles externally, one is tangent internally, three are tangent to two externally and one internally, and three are tangent to two internally and one externally.

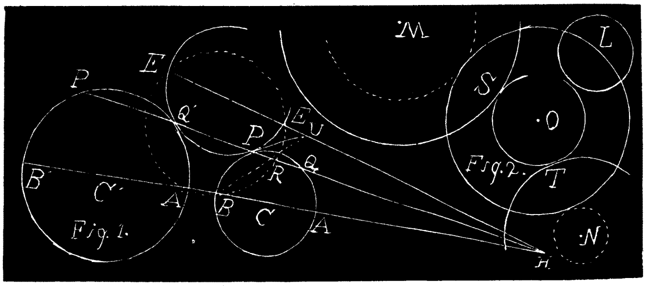


Fig. 1. Fig. 2.

In figure 1, to find a circle passing through a point E and tangent to two circles C, C'. Let H be the point where the external common tangent meets $C'C$ produced. Through $A'BE$ describe a circle cutting EH again in E'. Draw BR meeting HE in U, and draw UP tangent to C, then the circle through PEE' is the circle required.

Two tangents can be drawn from U.

If we had used the point where the internal common tangent cuts CC' we would have determined two other circles, four in all, satisfying the condition.

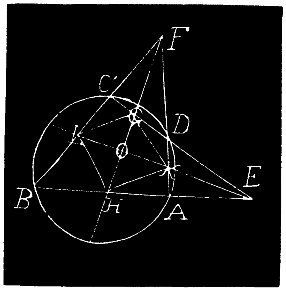
Also solved by *PROF. F. E. MILLER*, and *CHAS. C. CROSS*. Prof. Cooper D. Schmitt did not solve the problem but gave several references where solutions are given. Prof. J. Scheffer gave a short historical note on the problem.

In a future issue of the MONTHLY, we expect to publish a somewhat exhaustive discussion of this very interesting problem.

90. Proposed by G. B. M. ZERR, A. M., Ph. D., President and Professor of Mathematics in Russell College, Lebanon, Va.

The bisectors of the angles of the opposite sides (produced) of an inscribed quadrilateral cut the sides at the angular points of a rhombus.

Solution by G. I. HOPKINS, A. M., Professor of Mathematics in High School, Manchester, N. H.; J. K. ELLWOOD, A. M., Principal of Colfax School, Pittsburg, Pa.; J. W. SCROGGS, Principal of Rogers Academy, Rogers, Ark.; NELSON L. RORAY, Professor of Mathematics, South Jersey Institute, Bridgeton, N. J.; HENRY N. DAVIS, Providence, R. I.; ALOIS F. KOVARIK, Professor of Mathematics, Decorah Institute, Decorah, Ia.; and the PROPOSER.



In the triangles AEK and LEC, $\angle AEG = \angle LEC$, $\angle EAK = \angle LCE$.

$\therefore$ $\angle EKA = \angle ELC$. $\therefore$ $\angle DKL = \angle CLK$.

$\therefore$ FH is perpendicular to KL at its middle point. Similarly, EL is perpendicular to GH at its middle point.

$\therefore$ In the right triangles KOG, KOH, $KO=KO$, $GO=OH$.

$\therefore$ $KG=KH$. Similarly $KG=GL=LH$.

$\therefore$ $KGLH$ is a rhombus.

This problem was also solved in a similar manner by *E. T. BUSH* and *S. L. ROWAN*, of the Freshman Class of the University of Mississippi; *P. S. BERG*, *W. H. DRANE*, *F. R. HONEY*, *E. R. ROBBINS*, *B. F. SINE*, *J. SCHEFFER*, and *J. F. TRAVIS*.

CALCULUS.

70. Proposed by J. OWEN MAHONEY, B. E., M. Sc., Graduate Fellow in Mathematics in Vanderbilt University, P. O., Lynnville, Tenn.

Prove $\displaystyle\int_0^{\infty} \frac{\cos ax}{1+x^{2n}}dx = -i\frac{\pi}{2n}\sum_{r=1}^{n} \omega^{2r-1}e^{ai\omega^{2r-1}}$

where n is an integer, a is positive, and ω is $e^{i(\pi/2n)}$.

Solution by the PROPOSER.

Consider the integral $\displaystyle\int \frac{e^{iay}}{1+y^{2n}}dy$, where a is real and positive. The poles given by $y^{2n}=-1$ or $z=i^{1/n}=\cos(\pi/2n)+i\sin(\pi/2n)=e^{i(\pi/2n)}=\omega$ (say).

It is evident that all the roots of $y^n=i$ are given by ω^{2r-1}, where r may have the values 1, 2, 3, ...n.